CLASSIFICATION OF LINEAR RELAXATIONAL MODELS OF TWO-PHASE FILTRATION

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A classification of the linear relaxational models of two-phase filtration is given. Five types of nonequilibria are distinguished.

A model of nonequilibrium two-phase filtration, describing the process of displacement of oil by water, is studied in [1]. The model is based on the hypothesis that the departure from equilibrium can be taken into account by replacing the saturation in the phase permeabilities (or the Bakleya–Leverett function) by its effective value $\hat{s}$. At the same time the difference between s and $\hat{s}$ was assumed to be proportional to the rate of change of the saturation:

$$\hat{s} = s + \tau s_t' . \tag{1}$$

We shall analyze the relaxation filtration of a two-phase liquid from more general considerations.

1. For plane-parallel filtration with a constant velocity V = const we write down the system of equations

$$m \frac{\partial s}{\partial t} + V \frac{\partial}{\partial x} F(\hat{s}) = 0, \tag{2}$$

where

$$\hat{s} = \Phi(0) s + \int_0^\infty \frac{d\Phi}{dt}(t') s(t - t') dt. \tag{3}$$

Based on the principle of decay of the memory the following limitation is imposed on the kernel [2]: $\Phi(\infty) = \lim_{t\to\infty} \Phi(t) = 1$.

We shall study the propagation of small disturbances of the saturation along the sample, whose initial value is constant and equals $\bar{s}$. It is easy to see that $\hat{s} - \bar{s}$ is also small. Then, up to second-order infinitesimals, we obtain $F(\hat{s}) = F(\bar{s}) + F'(s)(\hat{s} - \bar{s})$. Taking into account (3), we write Eq. (2) for deviations of the saturation $s' = s - \bar{s}$ (in what follows we omit the prime)

$$\frac{1}{c_0} \frac{\partial s}{\partial t} + \Phi(0) \frac{\partial s}{\partial x} + \int_0^\infty \frac{d\Phi}{dt}(t') \frac{\partial s}{\partial x}(t - t') dt' = 0, \tag{4}$$

where $c_0 = VF'(\bar{s})m^{-1}$.

Next we shall study the solution of Eq. (4) with the following conditions:

initial

$$s(x, t) = 0, \quad t < 0, \tag{5}$$

and boundary

$$s(x_0, t) = f(t), \quad t > 0. \tag{6}$$

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It is easy to check that the solution of this problem also describes plane-parallel and spherically radial flow only with the x coordinate replaced by $r^2/2$ and $r^3/3$, respectively.

Applying to the relations (4) and (6) the Laplace–Carson transformation we obtain (* indicates the transform of the function):

$$\frac{ds^*}{dx}+\frac{s^*}{L^*(\sigma)}=0,\quad L^*(\sigma)=c_0\sigma^{-1}\Phi^*(\sigma),\ s^*(x_0,\ \sigma)=f^*(\sigma).$$

Determining s^* and returning to the original functions, we obtain

$$s(x,\ t)=(2\pi i)^{-1}\int\limits_{\gamma-i\infty}^{\gamma+i\infty}\sigma^{-1}f(\sigma)\exp\left(-L^{*^{-1}}(\sigma)(x-x_0)+\sigma t\right)dt.$$

We shall assume below that there exists an original function L(t). It is well known that to the asymptotics of the transforms in the limit $\sigma\to\infty$ ($\sigma\to 0$) there correspond asymptotics of the original functions on the right sides in the limit $t\to 0$ ($t\to\infty$) (Tauberian theorems). This is very convenient for studying the behavior of the solution.

2. One of the features of the transient processes of relaxational filtration is the fact that the boundary-value problems for some types of kernels $\Phi(t)$ are no longer properly posed. In these problems a mismatch between the boundary conditions and initial conditions, in the sense of the existence of a continuous solution in the closed space [x, T], is not admissible.

The asymptotics of the solution (4)-(6) for short times has the form

$$s(x,\ t)=f(0)\exp\left(-L^{-1}(0)(x-x_0)\right),$$

and therefore, 1) if $L(0) = 0$, then $s(x, 0) = 0$, and a mismatch between the boundary and initial conditions is admissible, and 2) if $0 < L(0) < \infty$, then a nonzero saturation distribution is established "suddenly" initially, and a mismatch between the boundary and initial conditions is inadmissible.

In the case $0 < L(0) < \infty$ it is interesting to examine a detailed asymptotic expansion of the solution for short times. Expanding $s^*(x, \sigma)$ at a point at infinity $\sigma = \infty$ in powers of $\bar{\sigma} = \sigma^{-1}$ (which according to the Tauberian theorems corresponds to the point $t = 0$) and then transforming back to the original functions, we obtain

$$s(x,\ t)=\exp\left(-L^{-1}(0)(x-x_0)\right)\left\{f(t)+\left[\frac{d\ln f(\sigma)}{d\bar{\sigma}}-(x-x_0)\frac{d(L^{*^{-1}})}{d\bar{\sigma}}\right]_{\sigma=\infty}\int\limits_0^{\infty}f(t)\,dt+o\left(\int\limits_0^{\infty}f(t)\,dt\right)\right\}.$$

It is not difficult to show that the velocity of propagation of the saturation front is determined by the formula

$$c=\lim_{\sigma\to\infty}\sigma L^*(\sigma).$$

It is obvious that in the case $0 < L(0) < \infty$, $c = \infty$, and if $L(0) = 0$, then $c = dL/dt(0)$.

Next in the case of a finite rate of propagation of the front, by analogy to geometric optics [3] we obtain an expansion of the solution near the displacement front:

$$s(x,\ t)=\exp\left(-a(x-x_0)\right)\left\{f\left(t-c^{-1}(x-x_0)\right)+b(x-x_0)\int\limits_0^{t-c^{-1}(x-x_0)}f(t)\,dt+o\left(\int\limits_0^{t-c^{-1}(x-x_0)}f(t)\,dt\right)\right\},$$

while at the front itself at $t = c^{-1}(x - x_0)$ we have a relation for the jump in the saturation

$$[s]=s\left(x,\ c^{-1}(x-x_0)\right)=\exp\left(-a(x-x_0)\right)f(0),$$

which indicates exponential decay of the perturbations in the stratum, where

$$a=\lim_{\sigma\to\infty}\left(L^{*^{-1}}(\sigma)-c^{-1}\sigma\right)$$

is the decay coefficient and

$$b = \lim_{\sigma\to\infty} [(L^{*-1}(\sigma) - c^{-1}\sigma - a)\,\sigma].$$

3. The study of the asymptotics of the solution for short times enables, employing the procedure of [4], classification as a function of the values of L(0), c, and a of all possible types of linear relaxation models of filtering of a two-phase liquid. The classification leads to five types of models.

Type 1. L(0) = 0 and c = 0. The medium does not lead to perturbations, so that models of this type must be ignored as being physically meaningless. An example of such a model is

$$\hat{s} + \tau\frac{\partial\hat{s}}{\partial t} = s,\ \Phi^*(\sigma) = (1+\tau\sigma)^{-1},$$
$$\Phi(t) = (1+\exp(-t\tau^{-1}))\,\eta(t),$$

$$L^*(\sigma) = c_0\sigma^{-1}(1+\tau\sigma)^{-1},\ L(t) = c_0(t+\tau(1-\exp(t\tau^{-1})))\,\eta(t),$$
$$L(0) = 0,\ c = \lim_{\sigma\to\infty} c_0(1+\tau\sigma)^{-1} = 0,\ a = 0.$$

Type 2. L(0) = 0, 0 < c < ∞. Models of this type admit a mismatch between the boundary and initial conditions. The rate of propagation of the displacement front is finite, so that three cases can be studied depending on the value of the damping coefficient:

I. Equilibrium filtration a = 0:

$$\hat{s} = s,\quad \Phi^*(\sigma) = 1,\quad \Phi(t) = \eta(t),$$
$$L^*(\sigma) = c_0\sigma^{-1},\quad L(t) = c_0 t\eta(t),$$

where η(t) is the Heaviside switching-on function;

$$c = c_0,\quad a = c_0^{-1}\sigma - c^{-1}\sigma = 0,$$

whence it follows that the jump in saturation remains. We have the case of piston displacement according to the Bakleya–Leverett scheme.

II. 0 < a < ∞:

$$\hat{s} + \tau_1\frac{\partial\hat{s}}{\partial t} = s + \tau_2\frac{\partial s}{\partial t},$$
$$\Phi^*(\sigma) = (1+\tau_2\sigma)(1+\tau_1\sigma)^{-1},$$
$$\Phi(t) = (1-(1-\tau_1^{-1}\tau_2)\exp(-t\tau_1^{-1}))\,\eta(t),$$
$$L^*(\sigma) = c_0\sigma^{-1}(1+\tau_2\sigma)(1+\tau_1\sigma)^{-1},$$
$$L(t) = c_0(t+(\tau_2-\tau_1)(1-\exp(-t\tau^{-1})))\,\eta(t),$$
$$c = c_0\tau_1^{-1}\tau_2 < \infty,\quad a = c_0^{-1}(\tau_2-\tau_1)\,\tau_2^{-2},$$

and there exists a jump in the saturation according to the law

$$[s]_x = \exp(-c_0^{-1}(\tau_2-\tau_1)\,\tau_2^{-2}(x-x_0))[s]_{x=x_0}.$$

III. a = ∞:

$$\hat{s} + \tau_1\frac{\partial^{1/2}\hat{s}}{\partial t^{1/2}} = s + \tau_2\frac{\partial^{1/2}s}{\partial t^{1/2}},\quad \Phi(\sigma) = (1+\tau_2\sqrt{\sigma})(1+\tau_1\sqrt{\sigma})^{-1},$$

$$\Phi(t) = \eta(t)(\pi t)^{-\frac{1}{2}}\int_0^\infty (1-(1-\tau_1^{-1}\tau_2)\exp(\tau_1^{-1}\tau)\exp(-\tau_2(4t)^{-1}))\,d\tau;$$

$c = c_0\tau_1^{-1}\tau_2$, like in the preceding model, and $a = \lim_{\sigma\to\infty}(\tau_2 - \tau_1)(\sigma^{-\frac{1}{2}} - \tau_1)^{-2}$ – the disturbances decay instantaneously.

Type 3. $L(0) = 0$, $c = \infty$. For models of this type a mismatch between the boundary and initial conditions is also admissible. The velocity of propagation of the perturbation front is infinite, but at infinity the perturbations are infinitesimal. An example of models of this type is:

$$\hat{s} = s + \tau \frac{\partial^{1/2} s}{\partial t^{1/2}},$$

$$\Phi^*(\sigma) = 1 + \tau\sqrt{\sigma},\ \Phi(t) = (1 + k(\pi t)^{-\frac{1}{2}})\,\eta(t) \quad (k = \text{const}),$$

$$L^*(\sigma) = c_0(1 + \tau\sqrt{\sigma})\,\sigma^{-1},\quad L(t) = c_0(t + 2k\tau\sqrt{t\pi^{-1}})\,\eta(t),$$

$$c = \lim_{\sigma\to\infty}(1 + \tau\sqrt{\sigma}) = \infty.$$

Type 4. $0 < L(0) < \infty$, $c = \infty$. Models of this type do not admit a mismatch between the boundary and initial conditions. The rate of propagation of the saturation front is infinite. Such models are not suitable for studying the full transient process, but they can be employed for not very sharp changes in the saturation, for example, as intermediate asymptotics. An example of this model, proposed by Barenblatt and Vinnichenko [1], is:

$$\hat{s} = s + \tau\frac{\partial s}{\partial t},\quad \Phi^*(\sigma) = 1 + \tau\sigma,\quad \Phi(t) = 1 + \tau\delta(t),$$

$$L^*(\sigma) = c_0\sigma^{-1}(1 + \tau\sigma),\quad L(t) = c_0(t + \tau)\,\eta(t),$$

where $\delta(t)$ is a Dirac delta function and $\eta(t)$ was determined above, $0 < L(0) = c_0\tau < \infty$, the saturation distribution $s(x, t) \underset{t\to 0}{\sim} f(0)\exp(c_0^{-1}\tau^{-1}(x - x_0))$ is established initially.

Type 5. $L(0) = \infty$, $c = \infty$. Models of this type also do not admit a mismatch between the boundary and initial conditions, since initially a constant saturation distribution $s(x, t) \underset{t\to 0}{\sim} f(0)$ is established "suddenly." An example of a model of this type is:

$$\hat{s} = s + \tau\frac{\partial^2 s}{\partial t^2},\quad \Phi^*(\sigma) = 1 + \tau\sigma^2,\quad \Phi(t) = (1 + \tau\delta'(t)),$$

$$L^*(\sigma) = c_0\sigma^{-1}(1 + \tau\sigma^2),\quad L(t) = c_0(t + \tau t^{-1})\,\eta(t).$$

The medium seems to exhibit infinite conductivity. Models of this type should not be employed to study the full transient process, but only as an intermediate asymptotics.

NOTATION

s, saturation of water; $\hat{s}$, effective saturation; t, time; $\bar{s}$, initial saturation of the water; τ, characteristic time for establishing equilibrium; $F(s)$, a function expressing the ratio of the moduli of the velocity of the water and the total velocity; V, total flow velocity; m, porosity; $\Phi(t)$, relaxation kernel; x, spatial coordinate; x_0, a boundary point; $f(t)$, saturation distribution fixed at the boundary; r, a variable radius; σ, Laplace–Carson transformation parameter; $L^*(\sigma)$ and $L(t)$ complex linear size in the transform plane and its original function; c_0, constant; c, velocity of propagation of the saturation front; and b, τ_1, and τ_2, constants.

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BEHAVIOR OF THE REFLECTION COEFFICIENTS OF TEXTOLITES UNDER LASER HEATING

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The reflection coefficients at the wavelengths 0.63, 1.15, and 10.6 μm of composite materials, heated in air by continuous CO_2 laser radiation, were determined experimentally.

Composite materials based on organic and glass fillers and polymer resin binders are being increasingly employed in modern technology. The most promising instruments for working these materials are the technological CO_2, YAG, and argon lasers, whose radiation can heat, melt, and evaporate the most heat-resistant materials.

The main parameter determining the efficiency of heat transfer from radiation to a nontransparent material with laser heating is the reflection coefficient. As a result interest in determining the reflective properties of composite materials in the spectral regions of lasing of technological lasers has increased [1-3]. Previously performed measurements of the reflection coefficients R_λ at wavelengths of 0.63, 1.15, and 10.6 μm of commercial textolites (Getinaks, textolite, glass-textolite), heated by continuous CO_2 laser radiation, showed that at the starting stage of irradiation irreversible thermochemical transformations (pyrolysis and charring), which decrease the reflection of short wavelength radiation (1.15 and 0.63 μm) and increase the reflectivity at the center (10.6 μm) of the IR region of the spectrum [4, 5], occur in the surface layer of the indicated materials. The temperature dependences of the reflection coefficients of cokes, forming on the surface of polymer composites under quite prolonged heating and fixed density of the incident laser flux, were obtained. However for thermophysical calculations of the heating and thermal destruction of the composite materials it is necessary to have information about the changes in the reflection coefficients and the temperature of the heated zone at the initial period of irradiation as a function of the incident flux density of the laser radiation. To this end, in this work we studied the behavior of the reflection coefficients of organic fiber plastics of the Getinaks and textolite PTK types as well as STK glass textolite as a function of the density q of the continuous CO_2 laser radiation flux incident on them. The textolites studied are polymer composite materials. Grade I Getinaks (GOST 2718-66) is a layered material, pressed from electrical insulation paper, permeated with phenol formaldehyde resin of the resol type. The construction textolite PTK (GOST 5-72) is prepared from cotton fabric, which, like the Getinaks, is permeated with resol resins based on phenol. The fill for the STK glass textolite (GOST 126-52), which exhibits an elevated heat resistance, consists of É glass cloth, and the binder is KO-926 silicone resin. These materials are employed for pipes, valves for chemical equipment, pinions, slide bearings, and other load-bearing components [6].

The measurements were performed on a setup which permits recording simultaneously the He–Ne and CO_2 laser radiation fluxes reflected from the samples and the temperature of the surface of the target in the heating zone, using the procedure described in [4]. The true temperature of the surfaces of the samples was determined from measurements of the intensity of the characteristics thermal radiation of the target and of the reflected radiation of the He–Ne laser employed as the illumination source at wavelengths of 0.63 or 1.15 μm. The errors in the temperature measurements were evaluated using the formula (2) [4] and did not

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